



Managing resilience and Viability: From formal concepts to practical application

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► **Examples of resilience**

- ❑ Can the water of a lake spoiled by nitrates input recover its purity ?
- ❑ Can a species recover from the introduction of a new predator in its environment ?
- ❑ Can an economy recover from a financial crisis ?



▶ **Formal definitions of resilience**

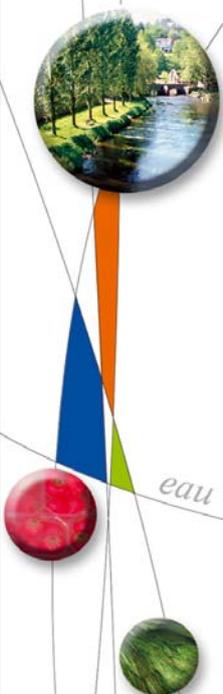
- ❑ Pimm (1984), resilience is the ability of a system to resist disturbance and the rate at which it returns to equilibrium following a disturbance.
- ❑ Gunderson and Holling (2002), resilience is the capacity of a system to undergo disturbance and maintain its functions and controls. Loss of resilience is associated with slow dynamics in a region that separates domains of attraction (Holling 1973).
- ❑ In the case of individual-based models and cellular automata, resilience is studied with simulations as the time needed after some kind of disturbance to return to its original state (Ortiz and Wolff 2002) or to reach a certain percentage of the previous abundance (Matsinos and Troumbis 2002)



▶ **Outline**

- ❑ Resilience as ability to come back to a « good » attractor
- ❑ Viability theory (J.P. Aubin)
- ❑ Resilience defined from viability theory (S. Martin)
- ❑ Practical applications ?





• Resilience as the ability to come back to a « good » attractor

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▶ **Simple example**

- ❑ Suppose that farmers have two possible activities *A* and *B* on a territory.
- ❑ When an activity is spread and attractive, it tends to spread more
- ❑ The public authority wants to keep some diversity among the techniques (to avoid that all farmers do the same)



▶ Mathematics

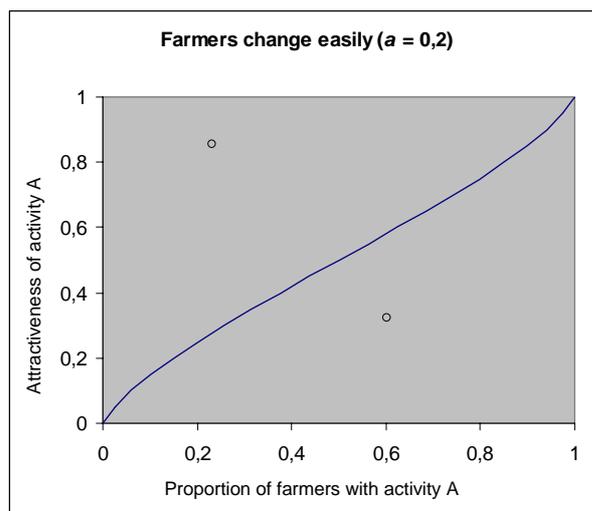
- Call σ_A the proportion of farmers doing A, $\sigma_B = (1 - \sigma_A)$ being the proportion of farmers doing B.
- Call s the attractiveness of A (compared with B)
- The evolution of the proportion of farmers with activity A is:

$$\frac{d\sigma_A}{dt} = \sigma_B \sigma_A^a s - \sigma_A \sigma_B^a (1 - s)$$

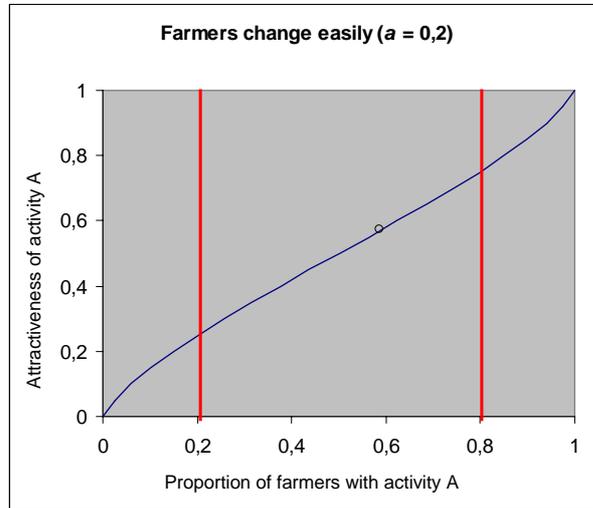
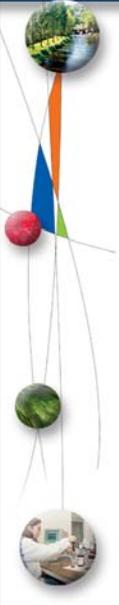
- If $a < 1$ then change is easy, if $a > 1$ change is difficult
- We suppose that the « good » diversity is:

$$0.2 \leq \sigma_A \leq 0.8$$

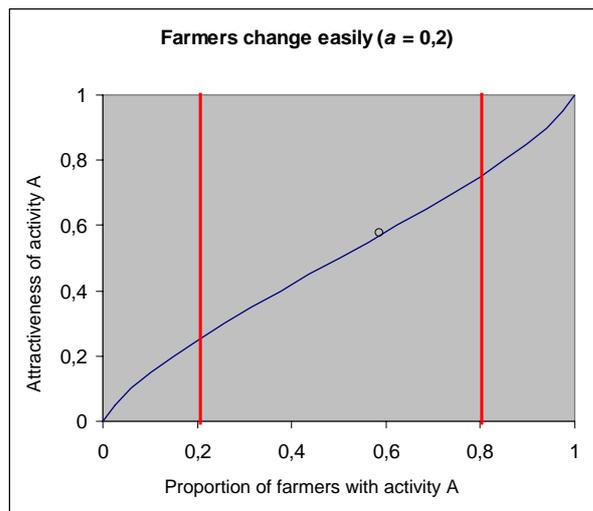
▶ Farmers can change easily ($a = 0.2$)



► **Farmers can change easily ($a = 0.2$)**



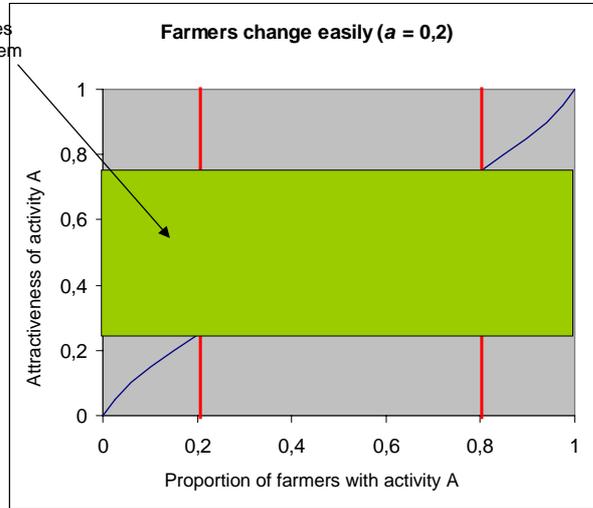
► **Farmers can change easily ($a = 0.2$)**



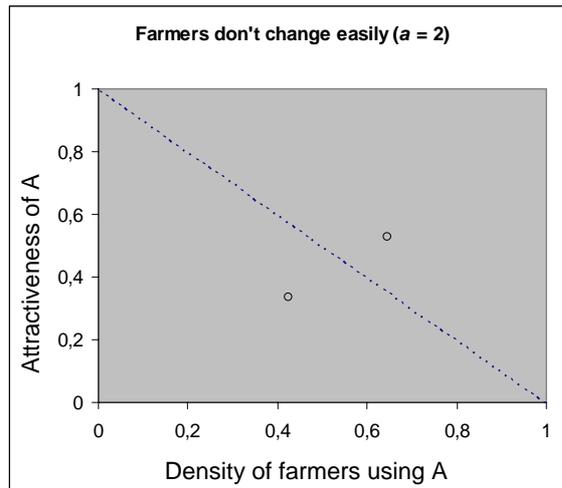
► **Farmers can change easily ($a = 0,2$)**



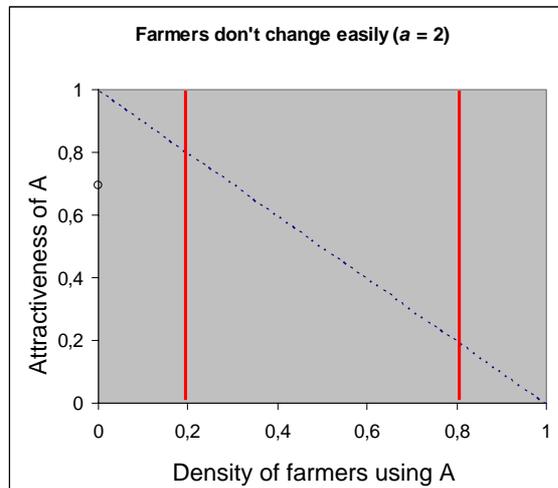
Perturbed states where the system is resilient



► **Farmers don't change easily ($a = 2$)**



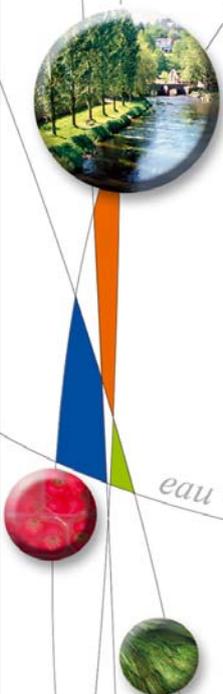
▶ *Farmers don't change easily ($a = 2$)*



The system is not resilient at all !

▶ *Is-it possible to manage resilience ?*

- In this standard view, this question is only about designing more or less resilient systems: enlarging the attraction basins.
- However, if you suppose one can act on the system and modify its trajectory, then depending on the choice of the policy of action, the system can be more less resilient. Examples:
 - the resilience of a patient's health depends on the choice of the medical treatment
 - the resilience of economic growth to crisis depends on the policy of its government
- Viability theory helps to define resilience in this more general context



Viability theory (JP Aubin 1991)
and Resilience

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▶ **Adaptation of the example**



- We suppose that a public body can modify the attractiveness s of technique only by a value ds such that:

$$-0.1 \leq ds \leq 0.1$$
- The viability problem becomes: How to find a policy of action $ds(t)$ on the attractiveness of technique A such that the diversity of techniques is always maintained (viability constraint set K):

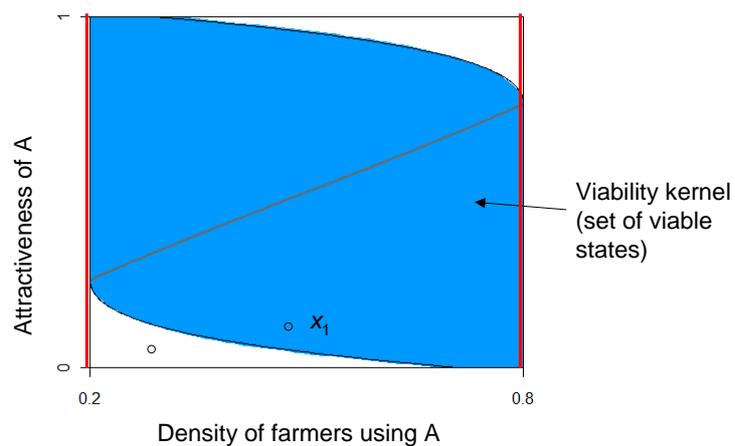
$$0.2 \leq \sigma_A \leq 0.8$$



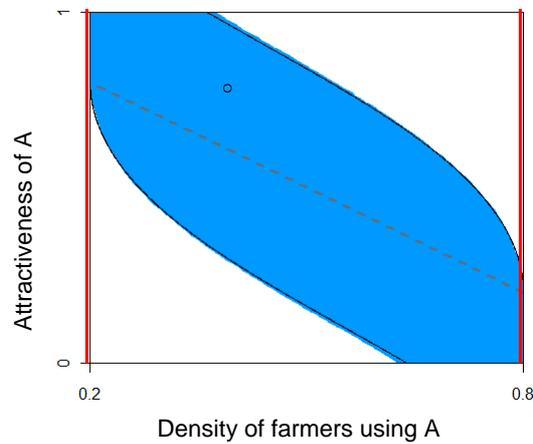
▶ Viability kernel

- Main concept of viability theory is the viability kernel, noted $Viab(K)$
- $Viab(K)$ is the biggest subset of K such that from any x_0 in $Viab(K)$, there exists an action keeping the system inside $Viab(K)$ at the next time step.

▶ Viability kernel for easy change ($a = 0.2$)



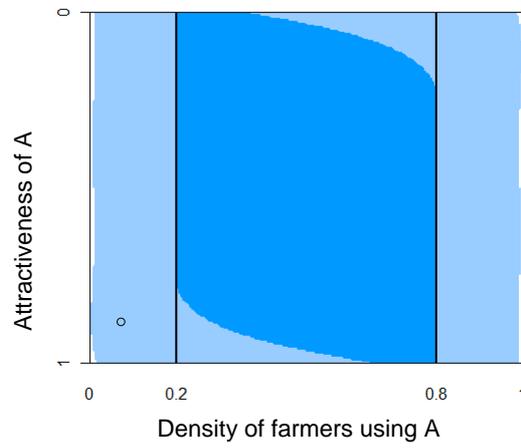
▶ **Viability kernel for non easy change ($a = 2$)**



▶ **From viability to resilience (S. Martin 2004)**

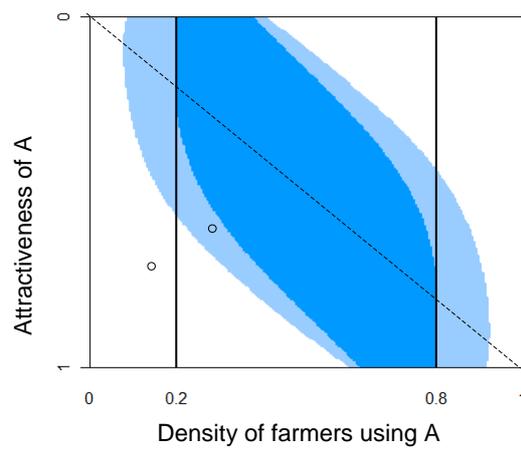
- Idea: the system is resilient for a perturbation, if after the perturbation, there exists a policy of action leading the system back to the viability kernel

► **Resilience for easy change ($a = 0.2$)**



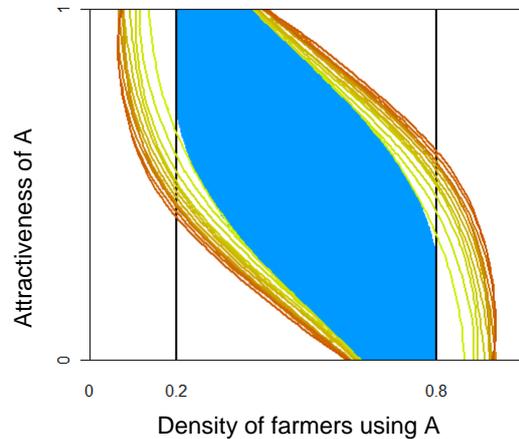
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► **Resilience for non easy change ($a = 2$)**



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▶ Resilience indices and resilient policies

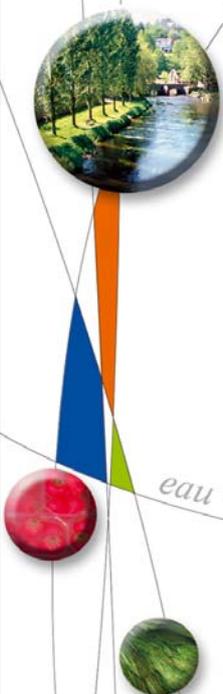


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▶ Summary

- Dynamical system, on which action is possible
- A set of viability constraints is identified, expressed as a subset of state space
- Solve viability problem: Define the viability kernel, and associated viable policies
- Solve resilience problem: Define resilient states, and associated resilient policies, leading the system into the viability kernel

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• Conclusion: What practical lessons ?

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▶ **Advantages**

- No need for equilibrium to define resilience
- Extension to a case action on the system is considered
- Compute the policy of actions.





▶ **Limits**

- ❑ Need a mathematical model of the dynamics, including the effect of action and perturbations
- ❑ The computation of the viability kernel and then resilient state is exploding when the dimension of the state space increases.
- ❑ -> research to tackle these problems in PATRES EU project (NEST 043269)
<http://www.patres-project.eu>



▶ **General approach based on the concepts**

- ❑ Define the viability constraints or limits of acceptability (largely a political problem)
- ❑ Identify possible means of action and perturbations (political and technical)
- ❑ Identify the dynamics of the system, including effects of perturbations and of the means of actions (technical)
- ❑ Identify policies of actions for (technical):
 - Maintaining viability
 - Restoring viability after crisis (perturbations)
- ❑ Discuss the results (acceptability of policies) and loop again.

▶ **Framework for reflection on concepts**

□ A. Stirling presentation:

| | Control (action on the perturbation or stress to keep system OK) | Response (action on the system after perturbation or stress) |
|---------------|---|---|
| Perturbations | Stability Perturbation is part of the system dynamics. Viability problem | Resilience Resilience problem, (linked to a Viability problem) |
| Stresses | Durability Stresses (long term tendencies, should be integrated into the system dynamics). Viability problem | Robustness |